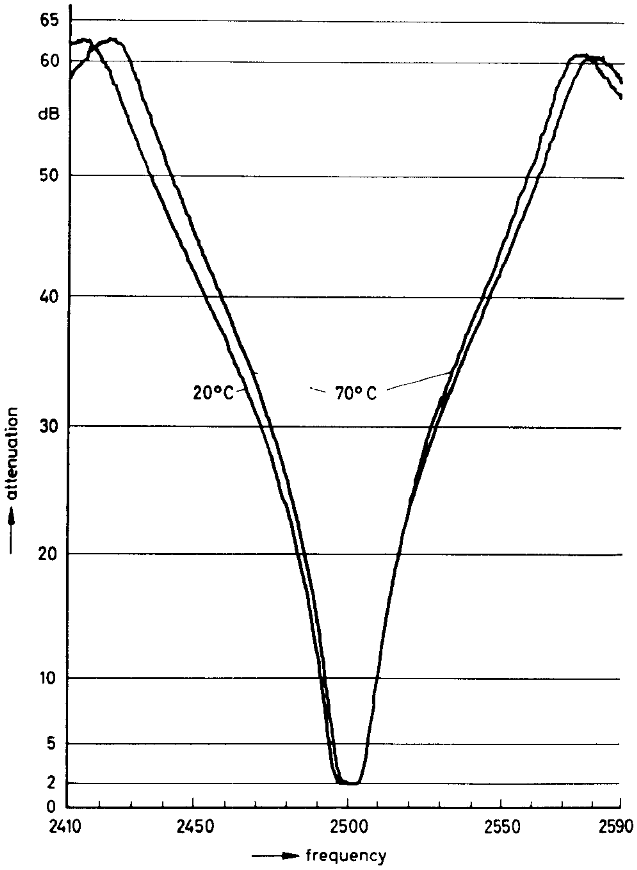

Fig. 8. Effect of operating temperature on the attenuation poles; $\alpha_M = 48$ dB.

of α_p can be expected up to 6 dB below the actual crosstalk values. This is confirmed by the results for high α_M values in Table I and explains the deviation from (7). An asymmetry of the level of α_p also partly arises if the crosstalk of each of the two YIG-coupling sections is different and if α_M is different at the pole frequencies.

Due to the crosstalk, the upper limit of α_M for applications will be of the order of 55 dB; the lower limit is mainly given by the required rejection level, but also by the reduced effectiveness according to (7). This is demonstrated in Fig. 6 for the case of $\alpha_M = 40$ dB. Measured values of the coupling level α_M at 2 GHz to 4 GHz as function of dimension A and the separation B between the center conductor of the input and output coaxial lines (Fig. 2) are given in Table II. These data were obtained using a 0.01-in Al_2O_3 substrate for the MIC and 0.2-mm diam center conductors; it is expected that the substrate thickness has a similar influence on α_M to dimension A. Higher values α_M could be realized by increasing the distance B between the input and output line.

A typical frequency dependence of α_M can be seen in Fig. 7. Due to the very smooth variation of α_M, tuning over an octave from 2–4 GHz was achieved without substantial changes in the pole attenuation characteristic. A filter of the type described was placed into a laboratory electromagnet and the permanent magnets were removed from the yokes. The measured data are given in Table III. The increasing deviation of the pole frequencies from values after (6) at the lower end of the frequency range can be explained by the increasing passband ripple that has been included in Table III. A realization of the additional coupling α_M in conventional tunable YIG filters should be possible by modification of the design of Fig. 2.

Finally, the effect of the operating temperature on the pole attenuation is shown in Fig. 8. The pole frequencies vary according to the 3-dB bandwidth reduction, as has been described before.

The realization of finite-pole frequencies in a two-stage YIG filter is of interest for applications that require a high rejection only at a small frequency band relatively close to the passband frequency, e.g., for image signal suppression with $|f_p - f_0|$ equal to twice the value of the intermediate frequency, instead of using a three-stage filter.

ACKNOWLEDGMENT

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Computer Analysis of Latching Phase Shifters in Rectangular Waveguide

FRED E. GARDIOL

***Abstract*—Latching phase shifters, consisting of a waveguide section containing a ferrite toroid, are widely used as digital steering elements in microwave array antennas. The theoretical determination of device performance cannot be obtained exactly, since these structures are inhomogeneous along both transverse directions.**

The present study presents an approximate method to evaluate phase shift and losses in the case of a rectangular toroid. An approximately equivalent structure (twin slab), for which an exact resolution method is available, is considered first. The changes due to the upper and lower sections of the toroid are then evaluated by means of a variational principle. Experimental results show good agreement with computed values for several practical cases considered. Finally, the range of validity for this approximate method is determined.

I. INTRODUCTION

Ferrite latching phase shifters are major components in modern phased-array radar systems; they generally consist of a rectangular waveguide containing hollow ferrite cylinders magnetized to remanence by means of thin conducting wires. Fig. 1 depicts a widely used configuration, a ferrite toroid of rectangular shape located at the center of the waveguide; the dimensions and the coordinate axes are also indicated on the drawing. The structure is inhomogeneous along both transverse directions $\bar{a}_x$ and $\bar{a}_y$; therefore, an exact analytical resolution for the electromagnetic fields and for the propagation characteristic is not feasible.

The microwave properties of this device can be determined to some extent from the analysis of the twin-slab phase-shifter structure shown in Fig. 2, which is homogeneous along the $\bar{a}_y$ direction. The transverse resonance method can then be used to determine the electromagnetic field distribution and the propagation coefficient [1]–[3]. Results for the twin-slab phase-shifter geometry have been used in the phased-array industry as a first-order approximation to predict the behavior of toroidal devices. However, differences in differential phase shift up to 20 percent or more have been observed between the theory for twin slabs and measurements taken on rectangular toroids. These differences can be either positive or negative; they depend on the material, the geometrical parameters, and the frequency. For instance, if a twin-slab structure is selected to yield a flat phase-shift characteristic versus frequency, this requirement will not be met by the corresponding toroidal device.

Several attempts were made to take into account the effect of the upper and lower sections of the ferrite toroid, leading to a rather amazing situation: for the two approaches published in the literature, the proposed corrections actually have opposite signs! Both of them are apparently based on sound theoretical considerations and

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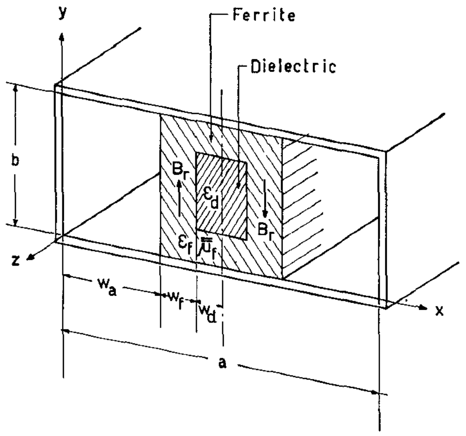

Fig. 1. Ferrite latching phase shifter in rectangular waveguide.

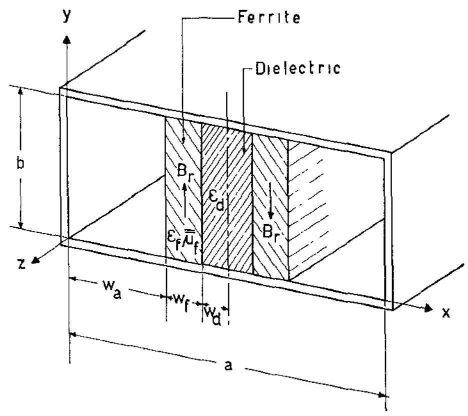

Fig. 2. Twin ferrite slab geometry.

are backed up by experimental evidence. In [2], it is simply assumed that the top and bottom ferrite members do not contribute any differential phase shift, since in those regions the RF magnetic field is not perpendicular to the remanent magnetization of the ferrite. A correction factor $1-2w_f/b$ reduces, accordingly, the differential phase shift. This factor was shown to be satisfactory for waveguides loaded with two toroids. It is generally too large for a single toroid. A more involved correction was proposed in [4], taking into account demagnetization within the ferrite domains and defining an effective internal magnetic field. The differential phase shift obtained is then larger than that of the twin-slab geometry, and good agreement is obtained with experimental data for empty rectangular toroids.

The very fact that two proposed corrections are diametrically opposed sheds serious doubts on the overall accuracy of the considerations used to derive them. Although each one has a certain range of validity over which some optimization of devices is feasible, the designer may feel somewhat puzzled: is the differential phase shift of a particular toroidal phase shifter going to be smaller or larger than that of the equivalent twin-slab device? The dielectric loading has a large effect on the electromagnetic field distribution within the device, and is therefore one of the major parameters of this problem. Surprisingly enough, little attention has been paid to this parameter in previous theoretical derivations.

An important prerequisite for the effective use of computer-aided design techniques is the availability of a suitable analysis method, which must describe accurately the performance of the device over a broad range of material parameters and dimensions. Any computation scheme requiring adjustable correction factors is limited to the range of parameters for which previous data are available, as any extension beyond this range may prove unreliable.

The approximate technique presented here allows one to predict with a good degree of accuracy the transmission properties of a latching phase shifter, containing one rectangular toroid of ferrite, when one specific condition is satisfied. Starting from the twin-slab geometry, a variational technique is used to determine the propagation constant in the doubly inhomogeneous structure. Comparing Figs. 1 and 2, the microwave fields in the two structures can be expected to be fairly similar, particularly so when $w_f \ll b$ and $\epsilon_f \simeq \epsilon_d$. The electromagnetic field distribution in the twin-slab structure can thus be used to determine an approximate solution for the ferrite toroid by means of the variational principle presented in [5]. A computer program was developed and computations were carried out for several geometries on which experimental data are available in the literature. Good agreement was obtained in most cases, and the range of validity of the method was determined.

II. Propagation Constant and Electromagnetic Fields in the Twin-Slab Geometry

A computer method for the analysis of a waveguide containing any number of lossy dielectric and transversely magnetized ferrite slabs extending across a rectangular waveguide was presented in [6]. It was later applied to the analysis of E-plane resonance isolators [5]. The same approach is used here to determine the propagation constant and the electromagnetic field distribution in the dual-slab structure. Due to symmetry, the plane $x=a/2$ is equivalent to a perfect magnetic conductor for the dominant quasi-TE_{10} mode, allowing thus to restrict the calculations to one-half of the waveguide cross section.

The relative tensor permeability of the ferrite magnetized to remanence in the $\bar{a}_y$ direction is given by

$$\boldsymbol{\mu}_r = \begin{bmatrix} \mu_r & 0 & jK \\ 0 & 1 & 0 \\ -jK & 0 & \mu_r \end{bmatrix} \tag{1}$$

where the diagonal μ_r and the off-diagonal K coefficients are functions of frequency and of the material parameters of the ferrite. This dependence is considered in Section IV.

Starting from the dimensions of the structure and the material properties of the dielectric and the ferrite, including their loss parameters, the computer program determines, by means of the search and interpolation procedure described in [6], the value of the complex propagation constants γ_s for both directions of propagation. The amplitude and phase of the electromagnetic field components E_y, H_x, and H_z are then calculated in the three waveguide regions. The peak power limitations of the device can be determined from these distributions. Electric discharge occurs when the electric field E_y exceeds the breakdown limit; nonlinear magnetic absorption occurs when the microwave magnetic field in the ferrite exceeds the spin-wave threshold field h_{crit}.

III. Effects of the Top and Bottom Parts of the Toroid

A variational principle for the propagation constant in lossy gyrotropic ferrites was presented in [5]:

$$\gamma = \frac{\int_A \eta \boldsymbol{H}_a \cdot \nabla_T \times \boldsymbol{E} + \boldsymbol{E}_a \cdot \nabla_T \times \eta \boldsymbol{H} - k\eta \boldsymbol{H}_a \cdot \boldsymbol{\mu}_r \cdot \eta \boldsymbol{H} - k\epsilon_r \boldsymbol{E}_a \cdot \boldsymbol{E}\, da}{\int_A \bar{a}_z \cdot (\boldsymbol{E} \times \eta \boldsymbol{H}_a - \boldsymbol{E}_a \times \eta \boldsymbol{H})\, da} \tag{2}$$

where

$\eta \quad = -j\sqrt{\mu_0/\epsilon_0} \simeq -j\ 377\ \Omega,$
$k \quad = \omega\sqrt{\mu_0 \epsilon_0},$
ϵ_r relative permittivity,

and where $\boldsymbol{E}_a$ and $\boldsymbol{H}_a$ are the associated fields, corresponding to the wave traveling in the opposite direction in a system characterized by $\boldsymbol{\mu}^T$ (reversal of the biasing magnetic field). Both integrals are taken over the waveguide cross section.

A correction for the top and bottom of the toroid can be determined from [5] assuming that the electromagnetic fields in the toroidal structure are approximately the same as those in the dual-slab geometry, which were determined in Section II. The introduction of the latter into [5] will yield a valid approximation for the propagation constant γ_t in the toroidal geometry, since variational principles are relatively insensitive to small errors in the field amplitudes. To simplify the computations, relation (2) is also applied to the twin-slab geometry, yielding in this case the exact value for the propagation constant γ_s in this structure. Subtracting the two expressions,

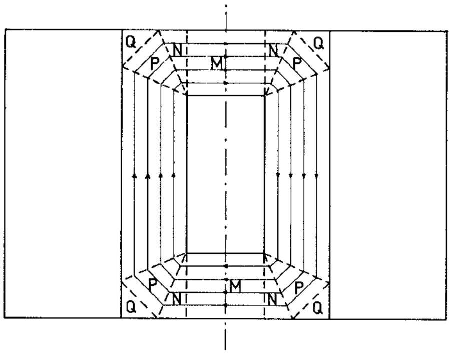

Fig. 3. Model used to evaluate the effect of the corners and of the top and bottom sides of the toroid (the arrows show the direction of dc remanent magnetization).

we obtain a perturbation equation:

$$\gamma_t - \gamma_s = \frac{-k \int_A \eta \boldsymbol{H}_a \cdot (\boldsymbol{\mu}_t - \boldsymbol{\mu}_s) \cdot \eta \boldsymbol{H} + (\epsilon_t - \epsilon_s) \boldsymbol{E}_a \cdot \boldsymbol{E} \, da}{\int_A \bar{a}_z \cdot (\boldsymbol{E} \times \eta \boldsymbol{H}_a - \boldsymbol{E}_a \times \eta \boldsymbol{H}) \, da} \tag{3}$$

where the subscript t refers to the toroidal structure and the subscript s to the twin slab.

A similar perturbation relation was obtained from comparison of Maxwell's equations in the two systems [7]. However, this approach does not utilize the error reduction contained in variational principles and would yield less accurate results than (3), particularly in the presence of large absorption. In the lossless system limit, the associated fields become complex conjugates and the two approaches are identical.

The field components in the twin-slab geometry are given by:

$$\begin{aligned} \boldsymbol{E} &= \boldsymbol{E}_a = \bar{a}_y E_y \\ \boldsymbol{H} &= \bar{a}_x H_x + \bar{a}_z H_z \\ \boldsymbol{H}_a &= -\bar{a}_x H_x + \bar{a}_z H_z. \end{aligned} \tag{4}$$

They are obtained in the manner described in Section II.

Introducing these field components into (3), we find for its denominator, taking symmetry into account:

$$4b \int_0^{a/2} E_y H_x \, dx. \tag{5}$$

The approximate model shown in Fig. 3, in which the toroid is divided into several regions, is utilized to evaluate the numerator. Only the regions labeled M, N, P, and Q provide nonzero contributions, as in all other regions $\boldsymbol{\mu}_t = \boldsymbol{\mu}_s$ and $\epsilon_t = \epsilon_s$. The dielectric contribution to (3) is given by:

$$\int_M (\epsilon_f - \epsilon_d) E_y^2 \, da. \tag{6}$$

The evaluation of the magnetic contribution is somewhat more involved: it becomes necessary to determine the ferrite permeability tensor when the magnetization is located in the x–y plane, making an angle θ with the $\bar{a}_y$ coordinate axis. In the x, y, z system it is then given by:

$$\boldsymbol{\mu}_f = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \mu_r & 0 & jK \\ 0 & 1 & 0 \\ -jK & 0 & \mu_r \end{bmatrix} \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \mu_r \cos^2\theta + \sin^2\theta & (\mu_r - 1)\cos\theta \sin\theta & jK\cos\theta \\ (\mu_r - 1)\cos\theta\sin\theta & \mu_r \sin^2\theta + \cos^2\theta & jK\sin\theta \\ -jK\cos\theta & -jK\sin\theta & \mu_r \end{bmatrix} \tag{7}$$

and then:

$$\boldsymbol{H}_a \cdot \boldsymbol{\mu}_f \cdot \boldsymbol{H} = -(\mu_r \cos^2\theta + \sin^2\theta) H_x^2 + \mu_r H_z^2 - 2jK\cos\theta H_x H_z. \tag{8}$$

For the twin slab, $\theta = 0$ in region N, P, and Q, while $\mu_r = 1$ and $K = 0$ in the dielectric region M. In the toroid, $\theta = \pm 90°$ in regions M and N, and $\theta = \pm 45°$ in region P. In the region Q, the ferrite is considered to be nonmagnetized: replacing the triangular sections Q by a dielectric of same permittivity does not change appreciably the differential phase shift [8], [9]. However, the ferrite in these regions still contributes magnetic loss. Device performance can be significantly improved by chamfering the corners; i.e., by removing region Q. This corresponds to letting $\epsilon_t = 1$ and $\boldsymbol{\mu} = \boldsymbol{I}$ within this region (see Section VI).

The integrals are evaluated numerically and the correction term $\gamma_t - \gamma_s$ is determined. Adding to the previously computed value of γ_s, the propagation constant γ_t for the ferrite toroid is obtained. The computations are carried out for both directions of propagation, and the differential phase shift is then determined.

IV. Determination of Ferrite Parameters

The best possible agreement between theory and experiment would of course be obtained by using measured values of μ_r and K in the determination of γ_t. As this information is not readily available, the designer must determine these parameters on the basis of formulas derived for suitable mathematical models. The ferrite used in remanent phase shifters is partially magnetized, i.e., the medium is inherently inhomogeneous, and the determination of average μ_r and K parameters therefore presents certain difficulties. This problem was considered by several authors who developed a number of different models (see bibliography of [10]). As the purpose of the present study is not to determine which is the best model, but rather to demonstrate that an adequate prediction is feasible with available material data, the following simple model will be used (it is understood that better predictions could possibly be obtained by using more elaborate models):

$$\mu_r = 1 - j \frac{\pi M_s \Delta H_e}{(f/\gamma_g)^2 + (\Delta H_e/2)^2} (1 + R^2) \tag{9}$$

$$K = -\frac{4\pi M_s (f/\gamma_g) R}{(f/\gamma_g)^2 + (\Delta H_e/2)^2} \tag{10}$$

where $4\pi M_s$ is the saturation magnetization, ΔH_e the effective linewidth, f the frequency of operation, γ_g the gyromagnetic ratio, and $R = M_r/M_s$ the remanence ratio. The relation for μ_r was theoretically shown to represent a lower bound for the determination of losses [11].

In polycrystalline materials the effective ferrite linewidth ΔH_e, to be used in (9) and (10), is generally much smaller than the linewidth at resonance ΔH, which is broadened by porosity and local inhomogeneities [12]. A number of measurements of ΔH_e are reported in the literature [13], [14]; certain ferrite manufacturers presently measure it for all their low-loss materials [15].

An important possible source of error is the rather large spread of the remanent magnetization M_r, as this quantity appears to be very sensitive to small changes in the manufacturing process. Comparison of values measured by authors [16], [17] with catalog data show relative differences going up to 18 percent.

The comparisons given in the next section are therefore restricted to cases for which measured values of M_r are available. For similar reasons, the loss comparison was limited to the one case for which the experimental value of ΔH_e is given [4].

V. Comparison with Experimental Results

As the method developed here is an approximation, it is necessary to compare it with an experiment in order to determine its range of validity. Computations were carried out with several rectangular toroidal phase shifters for which experimental data have been published.

For the C-band phase shifter described by Ince *et al.* [16], the calculated value of 96°/in at 5.7 GHz compares very well with the measured value of 95°/in. For comparison the value for the corresponding (uncorrected) twin-slab structure is 118°/in (+23 percent), while use of the correcting factor of [2] yields 80°/in (−16 percent).

In the previous case the loading dielectric inside of the ferrite toroid had a permittivity of the same order as the ferrite. We now consider the case of a hollow ferrite toroid ($\epsilon_d \simeq 1$), for which comparative data are presented in Figs. 4 and 5. The correspondence between calculated and measured differential phase shifts is very good for a wide slot in the ferrite, though not quite as good when the width of the slot decreases. This is probably due to the fact that the

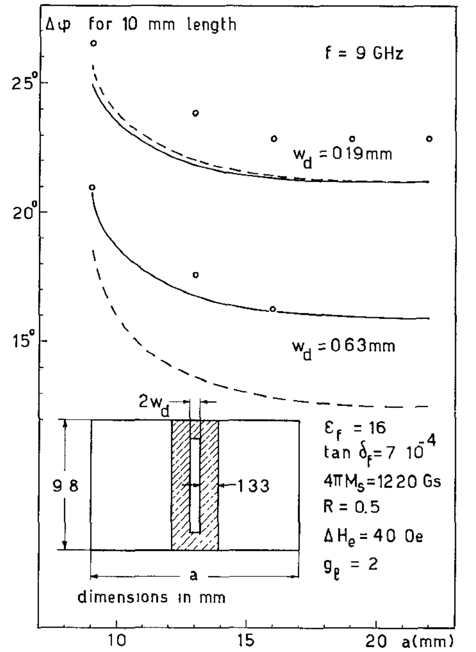

Fig. 4. Calculated and measured values of phase shift versus waveguide width. The circles correspond to experimental results published in [4], the solid lines to computed values (variational method), the dashed lines to the twin-slab approximation.

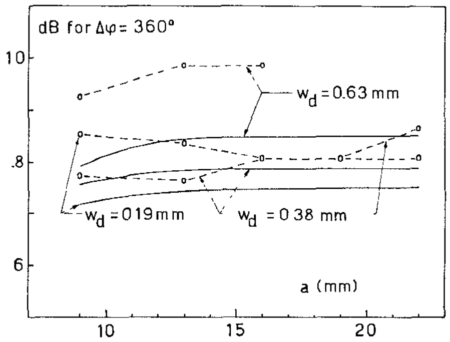

Fig. 5. Calculated and measured values of attenuation for 360° of differential phase shift for the structure of Fig. 4. Copper losses are determined with a perturbation method. The circles correspond to experimental results published in [4].

magnetization pattern is distorted by leakage between the two vertical legs of the toroid and is no longer similar to the one shown in Fig. 3. It must be noted that here the theoretical values for twin slabs are either equal to or lower than the ones for a rectangular toroid. Use of the correction factor of [2] would increase the discrepancy. The losses for this structure are shown in Fig. 5; the theoretical values are slightly lower than the measured ones, which is understandable since (9) is based on a lower bound.

Loading with high permittivity dielectric was recently reported [9], [17]. Comparative results for this case are presented in Fig. 6, showing a discrepancy between theory and experiment which increases when ϵ_d becomes large. In this case, the electromagnetic fields are strongly distorted, so that the assumption made previously is no longer valid when ϵ_d is larger than $\sim$25. A slightly better agreement is obtained by considering first the fields in a twin-slab structure containing a dielectric of average permittivity:

$$\epsilon_c = \epsilon_d + \frac{2w_f}{b}(\epsilon_f - \epsilon_d). \tag{11}$$

Even then, the approximation is not adequate for large values of ϵ_d. It is interesting to note that, for very large permittivity dielectrics, the measured phase shift is of the same order or even larger than the value calculated for the twin-slab geometry.

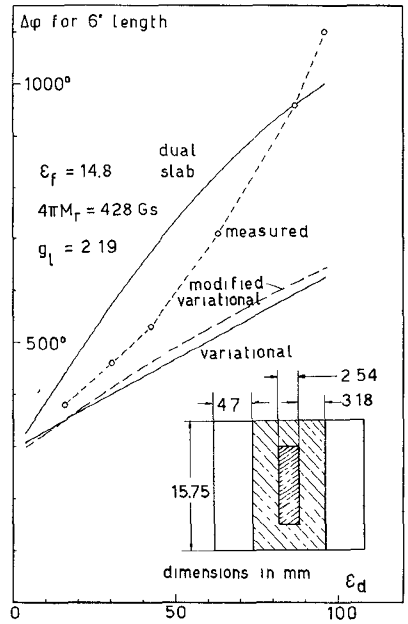

Fig. 6. Differential phase shift versus permittivity of the loading dielectric. The circles correspond to experimental results published in [9], the other lines to computed values.

VI. Chamfered Corners

The performance of rectangular toroid phase shifters can be somewhat improved by chamfering the corners; this increases the differential phase shift while losses are slightly reduced due to removal of ferrite material [8], [9]. Optimum results are obtained when the corners have a regular octagonal shape.

The method presented here allows us to study this effect: the material properties of region Q (Fig. 3) are simply replaced by those of air in the perturbation equation (3). Computations were carried out for the cases presented in [9], yielding increases in differential phase shift in the range 3–5 percent. The observed experimental increases are, however, in the 6–10-percent range, the discrepancy being probably also due in this case to distorsion of the RF fields in the corner region. Nevertheless, the qualitative agreement between theory and practice may help us understand the causes of the phase-shift increase.

VII. Conclusion

A computer technique to calculate the differential phase shift and losses of a remanent ferrite phase shifter was presented. The device consists of a rectangular ferrite toroid located at the center of a rectangular waveguide operating in the dominant TE_{10} mode. As the structure is nonuniform in two directions, it cannot be solved exactly by analytical means, and therefore approximate methods are necessary. A first approximation is obtained from the twin-slab geometry, for which an exact solution is available. The contribution of the top and bottom parts of the toroid is then evaluated by means of a variational principle. Comparison with several published experimental results shows a good agreement for many cases of practical interest. The range of application of this method is, however, limited by distortion of the fields in the corner regions, which occurs for large dielectric loading and for chamfered corners. This study did not consider end effects and matching techniques for the input and output ends of the loaded section, which would introduce a third nonuniformity and would be difficult to treat theoretically. In practice, however, experimental matching techniques are well established and quite adequate in general.

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Letters

Comments on "Modes of Propagation in a Coaxial Waveguide with Lossless Reactive Guiding Surfaces"

R. A. WALDRON

Many authors have attempted to simplify the study of the mode spectrum of a waveguide with a complicated wall structure by the use of a surface-impedance boundary condition, and the above paper[1] is a classic example of the exercise. The method depends on two assumptions—that it is proper to express a ratio between the tangential E and H fields as a boundary condition, and that such a ratio can be expressed unambiguously in terms of the form of the waveguide wall. That these assumptions are valid is always taken for granted by users of the method, including the present authors, but I have never seen a proof of their validity. Unless such a proof can be given, the results of calculations by the surface-impedance method cannot be trusted.

The results given by the authors in their Fig. 1 appear unusual, and do not agree with the results to the same problem obtained much more simply by applying perturbation theory [1] to the coaxial line, treated as a waveguide [1, sec. IV. F]. This suggests that the assumptions underlying the surface-impedance method require examination. That the method has been widely used does not establish the validity of the assumptions on which it is based.

I have made such a study in [2] where it is shown that there is no value of surface impedance that can be substituted into the characteristic equation obtained by the surace-impedance method that will make it identical with the true characteristic equation. It is also shown that, while approximate agreement between the characteristic equations can be obtained for *small* surface impedances, the value to be chosen for the surface impedance to secure this agreement is a complicated function of frequency and depends on the mode of propagation. Thus "surface impedance" is not, as has always been supposed, a property of the surface, and its value cannot be known until the solution to the problem under consideration is known. It is therefore of no help in solving a problem. It also follows that the assumption that any desired reactance can be realized is unfounded.

In short, the assumptions on which the surface-impedance method is based are invalid, and it is to this fact that the many strange results can be attributed that have been published by a number of authors. In view of the findings of [2], all results obtained by the surface-impedance method should be treated with caution.

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[1] R. K. Arora, S. Vijayaraghavan, and R. Madhavan, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-20, pp. 210–214, Mar. 1972.

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[1] R. A. Waldron, *Theory of Guided Electromagnetic Waves.* Princeton, N. J.: Van Nostrand-Reinhold, 1970, ch. 6.
[2] ——, *The Theory of Waveguides and Cavities.* London, England: Maclaren, 1967, ch. 5.

Reply[2] *by R. K. Arora*[3]

Waldron in his comment, as well as in [2] cited above, has assailed the use of surface impedance as a boundary condition.

The concept of surface impedance is several decades old and its use as a boundary condition has been made by many investigators. The conditions under which a surface may be characterized by an impedance-type boundary condition have been discussed by Senior [1] and Godzinski [2], and a further discussion of the usefulness of these conditions in solving practical problems is available in Barlow and Brown [3]. It is clear from [1] that it is possible, at least in principle, to devise structures with a prescribed value of surface impedance, and so the use of surface impedance as a boundary condition is justifiable on physical grounds. Though it is true that the surface-impedance description is not valid right at the discontinuity, experimental verification is obtained in microwave model experiments for ground-wave propagation [4], [5].

The surface-impedance method has proved to be of value in the solution of many problems of practical interest. No contradictions are

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